JEDEC STANDARD

Assessment of Average Outgoing Quality Levels in Parts Per Million (PPM)

JESD16B

(Revision of JESD16A, April 1995, Reaffirmed September 2008)

NOVEMBER 2017

JEDEC SOLID STATE TECHNOLOGY ASSOCIATION

Published by

3103 North 10th Street
Suite 240 South
Arlington, VA 22201-2107

PRICE: Contact JEDEC

Printed in the U.S.A.

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Introduction

Improvements in manufacturing technology and methodology have resulted in corresponding quality improvements for electronic devices. As a result, the traditional measure for reporting average quality levels in percent nonconforming needs to be replaced with a quantity more in line with the quality levels of today. That measure is parts-per-million or ppm. It is simply clearer to report estimated average device quality as 10 ppm, rather than the more cumbersome 0.001%. The ppm terminology applies to estimating the average outgoing quality (AOQ) level of a device, from lot acceptance results.

This standard was developed to provide a uniform method of measurement and calculation of average outgoing quality levels. Minimum sample sizes and a method for aggregating data are provided.

ASSESSMENT OF AVERAGE OUTGOING QUALITY LEVELS IN PARTS PER MILLION (PPM)

(From JEDEC Board Ballot JCB-17-18 formulated under the cognizance of JC-13 Committee on Government Liaison.)

1 Scope

This standard is intended to provide a uniform method of determining fraction nonconforming in finished devices and to provide a standardized definition of the quality index referred to as Average Outgoing Quality (AOQ). The method used here is primarily directed at devices whose production or procured volume is large enough, during some predefined sampling period, to give statistically meaningful information.

2 Application

This standard is intended to provide a method for the derivation and reporting of fraction non-conforming. The AOQ philosophy applies to the estimation of the average quality level of a product, not to lot acceptance plans. Since it is necessary to focus on accumulated lots to generate sufficient data for device quality characterization, the method in this standard should not be used to establish ppm levels for individual lots or to form the basis for determining acceptability of product on a lot-by-lot or batch-by-batch basis.

3 Reference publications

Publications referenced by this document are listed in Annex D.

4 Terms and definitions

For the purpose of this standard the following terms and definitions shall be used:

acceptance inspection: A sampling inspection or series of sampling inspections used to determine the suitability of a lot of material for shipment.

NOTE The accumulation of acceptance inspection data is used to determine average outgoing quality (AOQ).

accept number (c): The maximum number of nonconforming devices in the sample for which acceptance of the lot is allowed under the sampling plan.

average outgoing quality (AOQ): The expected population average nonconforming, in parts per million, estimated from a series of lots.

4 Terms and definitions (cont'd)

class: A categorization of similar characteristics for the purpose of reporting parts-per-million (ppm) nonconforming.

NOTE Examples of classes include functional (ppm1), electrical (ppm2), visual/mechanical (ppm3), hermetic (ppm4).

fraction nonconforming: The unknown noncomforming proportion of the total population of devices.

NOTE Estimates of fraction nonconforming are derived from samples.

group: A subdivision of a class based on inspection conditions or criteria, e.g., device type, product family, test temperature, sample size.

NOTE Lots that are members of the same group receive the same set of inspections in that group.

inspection: The assessment of a characteristic and its comparison to a standard.

NOTE Examples of inspections include low-temperature electrical test, room-temperature test, and visual inspection.

lot: An aggregate of devices from which the sample is selected.

lot acceptance rate (LAR): The ratio of lots inspected over a sample period.

nonconformance: A device characteristic that does not conform to an individual specified criterion.

nonconformity: A single device that has one or more nonconformances.

NOTE These nonconformities may be placed into classes for reporting purposes.

parts per million (ppm): The unit of measurement used to describe Average Outgoing Quality giving the number of nonconformities for each million units.

sample period: The period of time selected by the manufacturer to accumulate data for the calculation and reporting of average outgoing quality (AOQ) or lot acceptance rate (LAR).

5 Symbols

The following symbols are used in this standard:

Symbol	**Formula**	**Definition**
AOQ'		True value of the average outgoing quality.
AOQ		Estimate of AOQ'.
c		Sample plan acceptance number.
L		Total number of lots inspected.
LR		Total number of lots rejected.
LAR	1–(LR/L)	Lot acceptance rate.
m	S/L	Average sample size per lot
N		Total number of devices in lots inspected.
D		Total number of nonconforming devices found from all samples.
S	mxL	Total number of devices sampled.
p	D/S	Fraction of nonconforming devices found in all samples.
UB_1		Upper bound of a one-sided confidence interval for AOQ'.
LB_2		Lower bound of a two-sided confidence interval for AOQ'.
UB_2		Upper bound of a two-sided confidence interval for AOQ'.
σ^2		Variance of AOQ.
σ	σ^2	Standard deviation of AOQ.

6 Assumptions

The following assumptions are used by this standard. Users should confirm that these assumptions are met for the device whose quality is to be reported.

6.1 Attribute sampling inspection is being conducted on devices that have completed manufacturing processes affecting the criteria being reported.

6.2 Equipment must be calibrated according to ISO 10012-1, ANSI/NCSL Z540.1, ANSI/NCSL Z540.3, or ANSI/ASQC-M1. Accuracy and precision must be in accordance with manufacturer's internal documents or applicable procurement requirements. Statistical controls for the test process shall be in accordance with JESD557. For the purpose of estimating AOQ', test equipment can be used interchangeably provided the requirements of this paragraph are satisfied.

6.3 Lots of devices that fail acceptance inspection are reprocessed 100%, and all nonconforming devices are removed from the lot or the lot is removed from consideration for shipment and discarded. The nonconforming devices that were removed may or may not be replaced by conforming devices. In the case of c>0 sampling schemes, nonconforming devices found in the sample of accepted lots are removed from the sample and may or may not be replaced by conforming devices before the sample is returned to the lot for shipment.

6.4 All confirmed nonconforming devices observed during the first submission to acceptance inspection are included in the calculation of the total number of nonconforming devices found in samples (D) unless excluded according to 7.6. Data from resubmitted lots are not used in the calculation of AOQ.

6.5 Single, multiple or skip lot sampling may be used if appropriate (e.g., multiple temperature or hermeticity inspecting). Double counting of nonconforming devices is not allowed.

6.6 The total number of devices sampled (S) shall include only devices actually inspected.

6.7 AOQ values shall be reported for inspected characteristics only. Characteristics actually inspected and accompanying inspection results comprising the data for AOQ calculations shall be included in the manufacturing internal documentation and shall be available for review upon request.

6.8 Nonconformities that are not device-related shall not be included in the calculation of AOQ.

6.9 A single device with nonconformities in more than one class should be included only in the class that would have the most severe impact.

6.10 The estimator of AOQ given in equation (1) of 7.2 is appropriate if, for each lot, the sample size for the lot does not exceed 10% of the lot size.

6.11 The estimates of p and LAR shall be determined using the same samples.

6.12 All devices selected for acceptance inspection are assumed to be randomly sampled from the population being analyzed. (see Hahn and Meeker (1991) P10ff).

7 Procedures

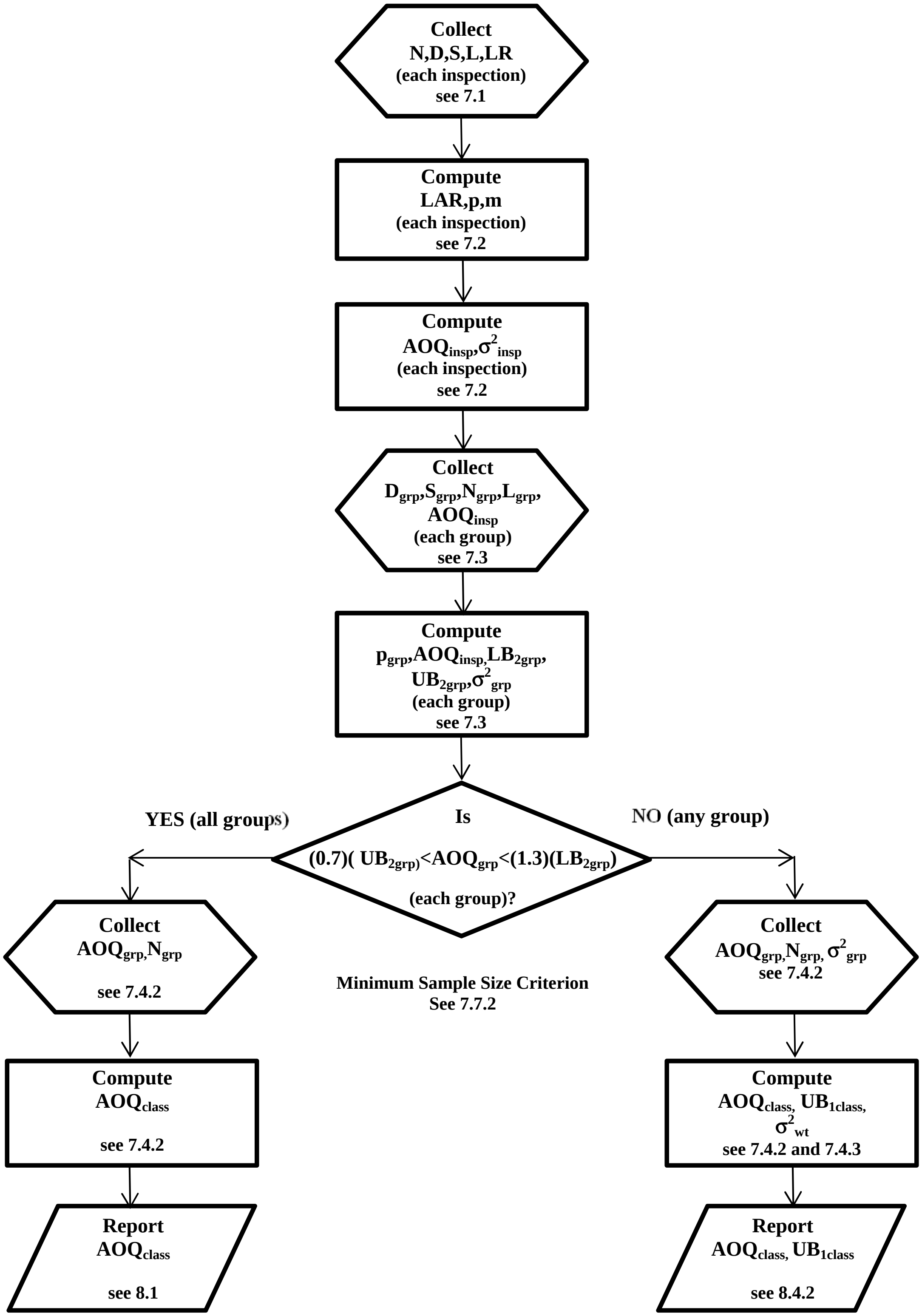

Figure 1 — Procedure Flow Chart

7 Procedures (cont'd)

7.1 Data accumulation for a single inspection

For all acceptance inspections during the sample period, sum each of the following independently and record for subsequent calculation:

- the number of observed nonconforming devices (D),
- the number of sampled devices(S),
- the number of devices in lots inspected (N), and
- the number of lots rejected (LR) and the number of lots inspected (L).

The sample period shall not exceed one year.

7.2 Computing AOQ and variance for a single inspection

7.2.1 Using the data totals from 7.1, the single inspection AOQ ($AOQ_{inspection}$) given by Schilling (1982) is:

$$AOQ_{inspection} = p \times LAR \times 10^6 \qquad (1)$$

7.2.2 When c = 0, an approximation of the variance σ^2 for AOQ, as defined in equation (1) is given in Annex C as:

$$\sigma^2 \approx \frac{(L - 1)p}{mL^3}\{L - 1 + 2mp(3 - 2L)\} \qquad (2)$$

If this approximation yields a negative result, then use the exact formula in Annex C.

7.2.3 If the sample size for any lot exceeds 10% of the lot size, then AOQ defined in equation (1) should be multiplied by $1 - \frac{S}{N}$. Equation (1) then becomes:

$$AOQ_{inspection} = p \times LAR \times \left\{1 - \frac{S}{N}\right\} \times 10^6 \qquad (1')$$

Similarly, equation (2) becomes:

$$\sigma^2 \approx \frac{(L - 1)p}{mL^2}\{L - 1 + 2mp(3 - 2L)\}\left\{1 - \frac{S}{N}\right\} \times 10^{12} \qquad (2')$$

7.3 Combining inspections within a group

For groups with more than one inspection, compute AOQ for each inspection within a group. Next compute the overall group AOQ by summing the AOQs for each inspection. It is important that lots within a group receive the same set of inspections. This means that a lot can belong to only one group within a class.

7.3.1 Example — One collection of devices is tested at three temperatures (low, room, and high). Let the AOQ for each test be denoted as follows:

$AOQ_{inspection1}$ = AOQ for low temperature test,
$AOQ_{inspection2}$ = AOQ for room temperature test,
$AOQ_{inspection3}$ = AOQ for high temperature test.

NOTE If devices fail more than one test in the group, they are only counted in the AOQ calculations for the first test they fail.

The overall group AOQ is calculated as follows:

$$AOQ_{group} = AOQ_{inspection1} + AOQ_{inspection2} + AOQ_{inspection3} \tag{3}$$

7.4 Combining groups within a class

To combine groups within a class such as electrical, mechanical, or hermetic, construct a weighted average according to the following guidelines.

7.4.1 For each group compute AOQ_{group}

7.4.2 Let $AOQ_{group\ i}$ be the AOQ for group i and $N_{group\ i}$ be the total number of devices in lots for group i. Also let there be k groups (e.g., i = 1 to k). Combine group AOQs within a class using weighted averages according to the following algorithm:

$$AOQ_{class} = \frac{\left(AOQ_{group1} \times N_{group1}\right) + \left(AOQ_{group2} \times N_{group2}\right) + \ldots + \left(AOQ_{groupk} \times N_{groupk}\right)}{N_{group1} + N_{group2} + \ldots + N_{groupk}} \tag{4}$$

7.4.3 Let σ_i^2 be the variance of the AOQ estimate for group i. The variance of AOQ_{class} is then found from equation (4) to be:

$$\sigma_{wt}^2 = \frac{(\sigma^2{}_{group1} \times N^2{}_{group1}) + (\sigma^2{}_{group2} \times N^2{}_{group2}) + \ldots + (\sigma^2{}_{groupk} \times N^2{}_{groupk})}{(N_{group1} + N_{group2} + \ldots + N_{groupk})^2} \tag{5}$$

NOTE This is not a weighted average of $\sigma^2{}_{group}$ values.

7.5 Combining classes

Classes such as electrical, mechanical, or other classes may be combined into a total AOQ by adding the class AOQs. Caution should be used when combining classes. The result is a worst case estimate of AOQ and can be erroneously inflated if the volume of product in each class is significantly different.

EXAMPLE — Let:

$AOQ_{electrical}$ = AOQ for the electrical class,
$AOQ_{mechanical}$ = AOQ for the mechanical class, and
AOQ_{other} = AOQ for some other class.

Compute the total AOQ (AOQ_{total}) for all classes by

$$AOQ_{total} = AOQ_{electrical} + AOQ_{mechanical} + AOQ_{other} \tag{6}$$

An example of how to calculate the AOQ for a given class which, when added to other classes can give the total AOQ, is shown in Annex A.

7.6 Data exclusion

All data from the first submission sampling inspection shall be included in the computation of AOQ, with the following exception:

- If the lot acceptance rate (LAR) is greater than 98% and
- within a given sample, the number of nonconforming devices found in the first submission is greater than or equal to the acceptance number plus three.

In this situation, the data from such a lot may be excluded from the computation of AOQ. See Hahn (1986). This exclusion rule effectively reduces the bias in estimating AOQ'. This can be verified by computer simulations.

7.6.1 The sample size and number of nonconformities associated with excluded lots shall not be included in the calculation of AOQ. However, LAR is not recalculated after excluding lots.

7.6.2 A log shall be maintained of all excluded lots. The log shall include the sample size, number nonconforming devices found in the sample, and lot size as a minimum. A technical explanation for the nonconformities may be added to the log.

7.7 Minimum total sample size

The total number of devices sampled (S) for the reporting period should be large enough that, with 90% confidence, the estimate of AOQ is within ±30% of AOQ'.

If the total units sampled do not meet the minimum requirements stated above and quantified in 7.7.1 or 7.7.2, then a **one-sided** upper 90% confidence bound for AOQ **shall** be computed using equation (7) and reported according to the format of 8.4.

7.7 Minimum total sample size (cont'd)

If there is more than one group within a class, the minimum sample size test must be applied to each group. If any group within the class fails the minimum sample size test, then a **one-sided** upper 90% confidence bound for the class AOQ **shall** be computed and reported using the weighted group AOQ of equation (4) and the weighted group variance of equation (5), as provided by equation (7), or equation (8).

An approximate upper 90% confidence bound given by Hahn and Meeker (1991) when D>0 is:

$$UB_1 = (AOQ)\left[\exp\left(\frac{1.282\sigma}{AOQ}\right)\right] \tag{7}$$

When D = 0 the upper 90% confidence bound (UB_1) given by Louis (1981) is:

$$UB_1 = (1 - \alpha^{(1/s)}) \times 10^6 \tag{8}$$

where $\alpha = 0.10$.

7.7.1 Minimum total sample size criterion (graphic method)

Figure B.1 in Annex B can be used to determine if the minimum total sample size requirement in 7.7 is achieved. The x-axis is AOQ in units of ppm. The y-axis is the total number of lots sampled (L). If the average single lot sample size (m) is between 100 and 200, then use the m = 100 minimum sample size line. If the average single lot sample size is greater than 200, then use the m = 200 minimum sample size line. If the average single lot sample size is less than 100, then use the numeric method in 7.7.2. If the intersection of AOQ and L falls above the appropriate minimum sample size line on the graph, then the minimum sample size has been achieved.

Example — Suppose the single lot sample size is m = 125, that the AOQ for these samples is 300 ppm, and that L = 2000 lots were inspected. Since m = 125 is between 100 and 200 use the m = 100 line. The intersection of the 300 ppm vertical line and the 2000 horizontal line falls above the m = 100 minimum sample size graph line on Figure B.1. Therefore, the minimum sample size is achieved.

7.7.2 Minimum total sample size criterion (numeric method)

A numeric minimum sample size test may be performed when desired. This procedure is used when results from the graph method are ambiguous (e.g., when the intersection of AOQ and L is very close to the appropriate minimum sample size line on Figure B.1) or when the average single lot sample size is less than 100.

The numeric method requires computing a two-sided confidence interval for AOQ' using equation (9), then checking to see if the AOQ estimate is within ±30% of each confidence bound. This numeric method is illustrated by example in annex B.

An approximate two-sided 90% confidence interval given by Hahn and Meeker (1991) when D>0 is:

$$[LB_2, UB_2] = \left[\frac{AOQ}{\exp\left(\frac{1.645\sigma}{AOQ}\right)}, (AOQ)\exp\left(\frac{1.645\sigma}{AOQ}\right)\right] \tag{9}$$

NOTE This confidence interval is not symmetrical around AOQ.

The minimum sample size requirement is satisfied if

$$(0.7)(UB_2) \le AOQ \le (1.3)(LB_2). \tag{10}$$

8 Reporting

8.1 To provide consistent reporting, this standard shall be specifically referenced if it is stated or implied that the AOQ has been derived in compliance with this standard.

8.2 Alternative estimators of AOQ may be used instead of the JESD16 estimator if comparable accuracy and precision can be demonstrated. Rules would need to be derived for the alternative estimator that govern the minimum sample size and confidence bounds. Conditions under which to apply the minimum sample size and confidence bound rules would apply as with the JESD16 estimator.

8.3 If nonconforming units are not found in the samples, it is recommended that an upper confidence bound be reported according to equation (11).

8.3.1 Report AOQ with an upper 90% confidence bound.

EXAMPLE — AOQ = 0, S = 50,000

$$\text{upper bound}_{ppm} = \left(1 - (0.10)^{\frac{1}{50,000}}\right) \times 10^6 = 46 \text{ ppm} \tag{11}$$

8.3.2 Format

AOQ = 0 with an upper 90% confidence bound of 46 ppm, based on a sample of 50,000 that contained no nonconforming units.

8.4 Reporting AOQ estimates when the minimum sample size is not achieved

8.4.1 EXAMPLE — Let:

D = 3,
S = 40,000,
LAR =.98,
L =320,
m = 125,
p = 3/40,000 = 7.5 x 10^5, and
c = 0.

From section 7.2:

$$\text{AOQ} = \left(\frac{\text{D}}{\text{S}}\right) \times \text{LAR} \times 10^6 = \left(\frac{3}{40{,}000}\right) \times 0.98 \times 10^6 \approx 74 \text{ ppm} \tag{12}$$

$$\sigma = 10^6 \sqrt{\frac{(\text{L-1})\text{p}}{\text{mL}^3}\left[\text{L - 1 + 2 mp (3 - 2L)}\right]}$$

$$= 10^6 \sqrt{\frac{(320 - 1)(7.5 \times 10^5)}{(125)(320)^3}\left[320 - 1 + 2\ (125)\left(7.5 \times 10^{-5}\right)\left(3 - (2)(320)\right)\right]} = 42.35$$

From section 7.7:

$$\exp(\text{t}) = \exp\left(\frac{1.645\sigma}{\text{AOQ}}\right) = \exp\left(\frac{(1.645)(42.35)}{74}\right) = 2.56 \tag{13}$$

$$\text{LB} = \left(\frac{\text{AOQ}}{\exp(\text{t})}\right) = \frac{74}{2.56} = 28.91.\ \text{AOQ} \times \exp(\text{t}) = (74)(2.56) \approx 74 \text{ ppm} \tag{14}$$

Expression (10) is not satisfied since AOQ = 74 is not greater than (0.7)(UB) = 132.757 or since AOQ = 74 is not less than (1.3)(LB) = 37.033. An upper 90% confidence bound will have to be computed and reported.

$$\text{UB}_1 = (\text{AOQ})\ \left[\exp\left(\frac{1.282\sigma}{\text{AOQ}}\right)\right] = 74\ \left[\exp\left(\frac{1.282(42.35)}{74}\right)\right]$$

$$\approx 154 \text{ ppm.} \tag{15}$$

8.4.2 Report format

Example: AOQ = 74 ppm with an upper 90% confidence bound of 154 ppm.

Annex A (informative) Combining group data and checking for minimum sample size

In the electrical class, a device can be inspected at up to three different temperatures. Define Group 1 to be all devices inspected at room temperature only. Define Group 2 to be all devices inspected at low, room, and high temperatures. Additional groups can be defined for other inspection combinations as necessary. Note that the way the groups are defined, it is not possible for a lot to belong to both groups. Table A.1 contains the data for the calculations in this section.

Table A.1 — Example data

Symbol	Group 1	Group 2	Definition
N	150,000	40,000	Total number of devices in lots inspected.
$D_{inspection1}$	n/a	0	Total number of nonconforming devices found from samples at inspection 1 (low temperature).
$D_{inspection2}$	2	2	Total number of nonconforming devices found from samples at inspection 2 (room temperature).
$D_{inspection3}$	n/a	3	Total number of nonconforming devices found from samples at inspection 3 (high temperature).
S	18,750	5,000	Total number of devices sampled.
L	150	40	Total number of lots inspected.
$LR_{inspection1}$	n/a	0	Total number of lots rejected at inspection 1 (low temperature).
$LR_{inspection2}$	1	1	Total number of lots rejected at inspection 2 (room temperature).
$LR_{inspection3}$	n/a	3	Total number of lots rejected at inspection 3 (high temperature).
m	125	125	Average sample size.
c	0	0	Sample plan accept number.
NOTE The number of lots (L) and the total sampled (S) for each inspection within a group is the same.			

Annex A (informative) Combining group data and checking for minimum sample size (cont'd)

For Group 1 there was only one inspection, therefore $AOQgroup_1 = AOQ_{inspection2}$.

EXAMPLE
See (1).

$$P_{inspection2} = \frac{D_{inspection2}}{S_{inspection2}} = \frac{2}{18750} = 1.0667 \times 10^{-4}$$

$$AOQ_{inspection2} = P_{inspection2} \times \left(1 - \frac{LR_{inspection2}}{L_{inspection2}}\right) \times 10^6$$

$$= (1.0667 \times 10^{-4} \times \left(1 - \frac{1}{150}\right) \times 10^6 \approx 106 \text{ ppm.} \qquad \text{(A.1)}$$

See (3).

$$\sigma = 10^6 \sqrt{\frac{(L-1)p}{mL^3}[L - 1 + 2mp(3 - 2L)]}$$

$$= 10^6 \sqrt{\frac{(150-1)(1.067 \times 10^{-4})}{(125)(150)^3}[150 - 1 + (2)(125)(1.067 \times 10^{-4})(3 - (2)(150))]}$$

$$\approx 73 \text{ ppm}$$

$$\sigma^2 = (73)^2 = 5315 \text{ ppm}^2. \qquad \text{(A.2)}$$

See (9).

$$\exp(t) = \exp\left(\frac{1.645\sigma}{AOQ}\right) = \exp\left(\frac{(1.645)(73)}{106}\right) = 3.10$$

$$LB = \frac{AOQ}{\exp(t)} = \frac{106}{3.101} \approx 34\text{ppm.} \qquad \text{(A.3)}$$

$$UB = (AOQ)(\exp(t)) = (106)(3101) \approx 329 \text{ ppm.}$$

The minimum sample size criterion

$(0.7)(UB) \leq AOQ \leq (1.3)(LB)$

which translates to

$(0.7)(329) \leq 106 \leq (1.3)(34) \Rightarrow 230 \leq 106 \leq 44$

is not satisfied.

Therefore, a one-sided 90% confidence bound will need to be computed and reported with AOQ_{class}. This will be done after the following analysis of Group 2.

Annex A (informative) Combining group data and checking for minimum sample size (cont'd)

For Group 2, there were three inspections; therefore, compute AOQ for each inspection and then sum them. See (3).

$$
\begin{aligned}
AOQ_{inspection1} &= \frac{D_{inspection1}}{S_{inspection1}} \times \left(1 - \frac{LR_{inspection1}}{L_{inspection1}}\right) \times 10^6 \\
&= \frac{0}{5000} \times \left(1 - \frac{0}{40}\right) \times 10^6 = 0 \text{ ppm}, \\
AOQ_{inspection2} &= \frac{D_{inspection2}}{S_{inspection2}} \times \left(1 - \frac{LR_{inspection2}}{L_{inspection2}}\right) \times 10^6 \\
&= \frac{2}{5000} \times \left(1 - \frac{1}{40}\right) \times 10^6 = 390 \text{ ppm}, \\
AOQ_{inspection3} &= \frac{D_{inspection3}}{S_{inspection3}} \times \left(1 - \frac{LR_{inspection3}}{L_{inspection3}}\right) \times 10^6 \\
&= \frac{3}{5000} \times \left(1 - \frac{3}{40}\right) \times 10^6 = 555 \text{ ppm},
\end{aligned}
$$

$$
\begin{aligned}
AOQ_{group2} &= AOQ_{inspection1} + AOQ_{inspection2} + AOQ_{inspection3} \\
&= 0 + 390 + 555 = 945 \text{ ppm.}
\end{aligned}
\tag{A.4}
$$

Since group 1 failed the minimum sample size criterion, the test is not necessary for group 2 since an upper 90% confidence bound has already been required by the minimum sample size test on group 1. The variance of AOQ_{group2} will need to be computed in order to calculate the weighted variance.

In this example, the minimum sample size test will be performed for illustrative purposes only. The first step is to use equation (3) to find the approximate standard deviation associated with AOQ_{group2}. For this purpose, we need the overall value of p for group 2, viz.,

$$
p = \frac{0+2+3}{5000} = \frac{5}{5000} = 0.001 \tag{A.5}
$$

Then (See (3),

$$
\begin{aligned}
\sigma &= 10^6 \sqrt{\frac{(L-1)p}{mL^3}\left[L - 1 + 2mp(3-2L)\right]} \\
&= 10^6 \sqrt{\frac{(40-1)(0.001)}{(125)(40)^3}\left[40 - 1 + (2)(125)(0.001)(3-(2)(40))\right]} \\
&\approx 310 \text{ ppm} \\
\sigma^2 &= (310)^2 = 96281 \text{ ppm}^2.
\end{aligned}
$$

$$
\exp(t) = \exp\left(\frac{1.645\sigma}{AOQ}\right) = \exp\left(\frac{(1.645)(310)}{945}\right) = 1.716. \tag{A.6}
$$

Annex A (informative) Combining group data and checking for minimum sample size (cont'd)

The lower and upper 90% statistical confidence bounds for the group 2 AOQ' are then:

$$\begin{aligned}
LB &= \frac{AOQ}{\exp(t)} = \frac{945}{1.716} \approx 551 \text{ppm.} \\
UB &= (AOQ)(\exp(t)) = (945)(1.716) \approx 1622 \text{ ppm.}
\end{aligned} \tag{A.7}$$

See (9).

The minimum sample size criterion

(0.7)(UB)≤ AOQ≤ (1.3)(LB)

which translates to

(0.7)(1622)≤945≤(1.3)(551) ⇒ 1135≤945≤716

is not satisfied.

The electrical class AOQ for this example is the weighted average of the groups. See (4).

$$\begin{aligned}
AOQ_{electrical} &= \frac{(AOQ_{group1} \times N_{group1}) + (AOQ_{group2} \times N_{group2})}{N_{group1} + N_{group2}} \\
&= \frac{(106 \times 150000) + (945 \times 40000)}{150000 + 40000} \\
&\approx 283 \text{ ppm.}
\end{aligned} \tag{A.8}$$

The weighted variance is:

$$\begin{aligned}
\sigma^2_{wt} &= \frac{(\sigma^2{}_{group1} \times N^2{}_{group1}) + (\sigma^2{}_{group2} \times N^2{}_{group2})}{(N_{group1} + N_{group2})^2} \\
&= \frac{(5315 \times 150000^2) + (96281 \times 40000^2)}{(150000 + 40000)^2} \\
&\approx 7580 \text{ ppm}^2. \\
\sigma_{wt} &= \sqrt{7580} \approx 87 \text{ppm.}
\end{aligned} \tag{A.9}$$

See (5).

$$\begin{aligned}
\exp(t) &= \exp\left\{\frac{1.282\sigma_{wt}}{AOQ_{electrical}}\right\} = \exp\left\{\frac{(1.282)(87)}{283}\right\} = 1.484. \\
UB_1 &= (AOQ_{electrical})(\exp(t)) = (283)(1.484) \approx 420 \text{ppm.}
\end{aligned} \tag{A.10}$$

See (7).

Electrical AOQ = 283 ppm with an upper 90% confidence bound of 420 ppm.

Annex B (informative) Derivation of the minimum sample size criterion

Derivation of the minimum sample size criterion Numeric Method (refer to 7.7.2).

This annex illustrates the derivation of the minimum sample size needed for the estimate of AOQ to be within ±30% of AOQ' with a 90% confidence. Definitions (reproduced from section 5, Symbols):

AOQ'	=	true value of average outgoing quality.
AOQ	=	estimate of AOQ.
UB_2	=	upper bound of a two-sided confidence interval for AOQ'.
LB_2	=	lower bound of a two-sided confidence interval for AOQ'.
S	=	total number of devices sampled.

The 30% bound can be written as:

$$\frac{|AOQ' - AOQ|}{AOQ'} \leq 0.30 \text{ or } 0.70 \times AOQ' \leq AOQ \leq 1.30 \times AOQ' \tag{B.1}$$

Using expression (9) a two-sided 90% confidence interval can be created for AOQ . Expressed mathematically

$$LB \leq AOQ' \leq UB \tag{B.2}$$

with a probability of 0.90. If AOQ is to be within ±30% of AOQ' with a 90% statistical confidence, then AOQ must be within ±30% of both UB and LB simultaneously. AOQ is unknown, but the interval [LB,UB] includes AOQ' with a probability of 0.90. This is written as

$$|AOQ-UB| \leq (0.30)(UB) \Rightarrow (0.7)(UB) \leq AOQ \leq (1.3)(UB) \tag{B.3}$$

and

$$|AOQ-LB| \leq (0.30)(LB) \Rightarrow (0.7)(LB) \leq AOQ \leq (1.3)(LB). \tag{B.4}$$

Combining (B.3) and (B.4) into a single expression gives

$$(0.7)(LB) \leq (0.7)(UB) \leq AOQ \leq (1.3)(LB) \leq (1.3)(UB) \tag{B.5}$$

From (B.5) note that (B.3) and (B.4) are simultaneously satisfied only if

$$(0.7)(UB) \leq AOQ \leq (1.3)(LB) \tag{B.6}$$

is true. Expression (B.6) is the minimum sample size criterion of 7.7.2. From expression (9) it follows that (0.7)(UB)≤(1.3)(LB) if and only if AOQ≥3147σ, so the minimum sample size criterion cannot possibly be passed unless AOQ≥5.3147σ. This would be an acceptable alternative to expression (B.6).

Annex B (informative) Derivation of the minimum sample size criterion (cont'd)

Graphical Method (Refer to 7.7.1)

Data for Graph B.1 were generated by using a computer search for the minimum value of L that satisfied the inequality

$$(0.7)(UB) \leq AOQ \leq (1.3)(LB)$$

for a given value of AOQ and m. LAR was fixed at 0.98. Values of LAR between 0.8 and 1 did not significantly affect the determination of L as long as AOQ was less than 1000 ppm. UB and LB are defined by expression (9). Extensions of graph B.1 to lower AOQ levels are:

AOQ	L for m = 100	L for m = 200
100 ppb	4,011,238	2,005,537
10 ppb	40,113,871	20,056,853

NOTE ppb is parts per billion.

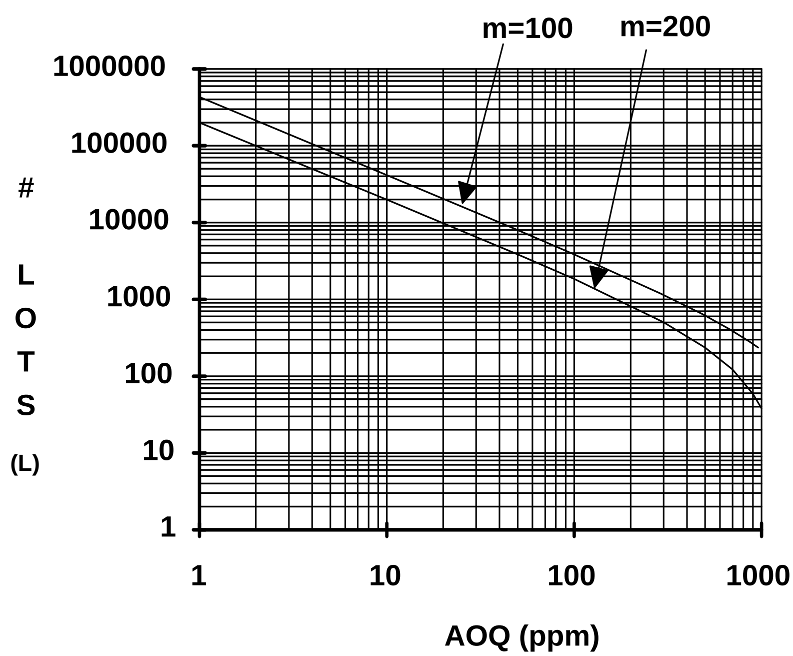

Figure B.1 — Graph for minimum sample size criterion

Annex B (informative) Derivation of the minimum sample size criterion (cont'd)

Graph B.2 is for information purposes. It is a graph of the value of AOQ that can be calculated when only one nonconforming device occurs in the sample. Smaller nonzero values of AOQ are not mathematically possible. The data were generated by dividing the cumulative sample size into 10^6.

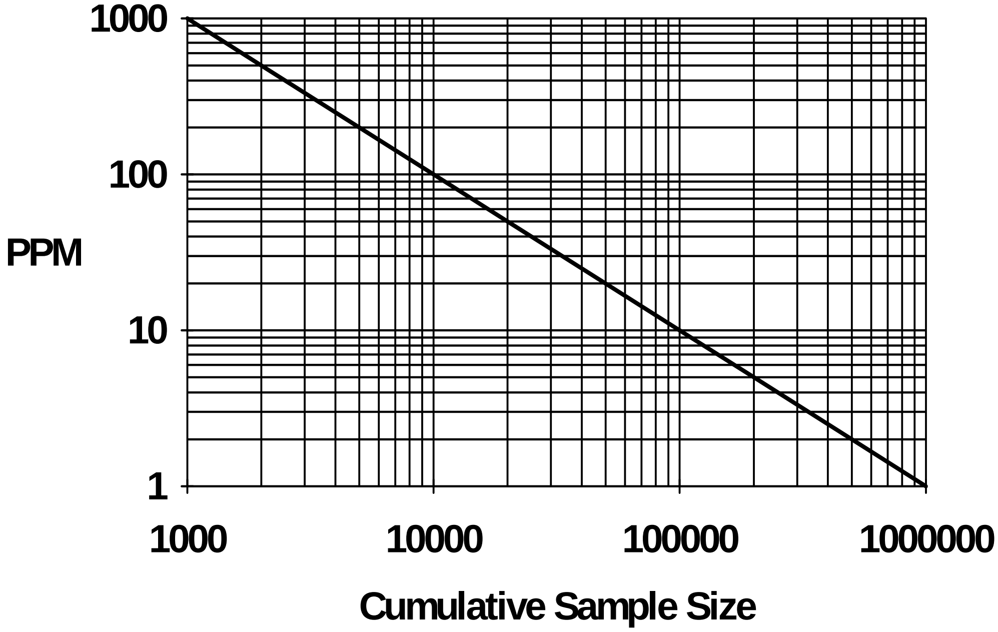

Figure B.2 — Minimum detectable PPM for a given sample size

Graphs B.3 and B.4 show the 90% upper confidence bounds for a selected number of lots sampled based on an LAR of 0.98. Graph B.3 corresponds to lot sample sizes of 100, and graph B.4 corresponds to lot sample sizes of 200. Equation (7) was used to generate the data for values of lots sampled L = 50, 100, 300, and 10000.

The numeric methods of equation (7) shall be used to compute upper confidence bounds. Graphs B.3 and B.4 should be used only for general indications of confidence bound values.

Example — Suppose 300 lots were sampled each with a sample size of 200 units. AOQ was computed to be 400 ppm. A corresponding approximate upper 90% confidence bound for AOQ' can be found on graph B.4 by locating the intersection of AOQ = 400 and the L = 300 line and finding the value of UB on the vertical axis to be 500 ppm.

Annex B (informative) Derivation of the minimum sample size criterion (cont'd)

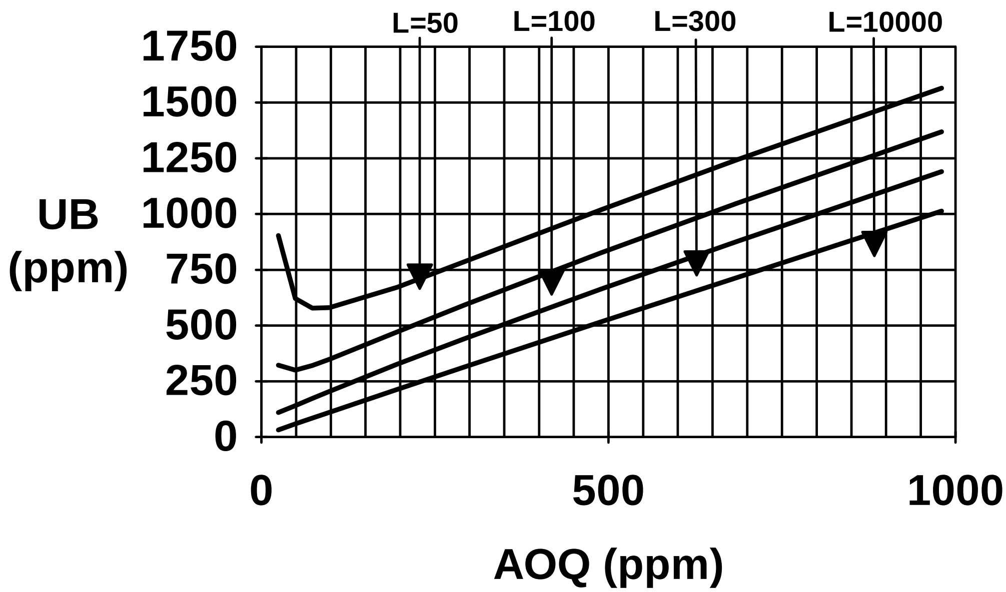

Figure B.3 — 90% upper bound, m=100

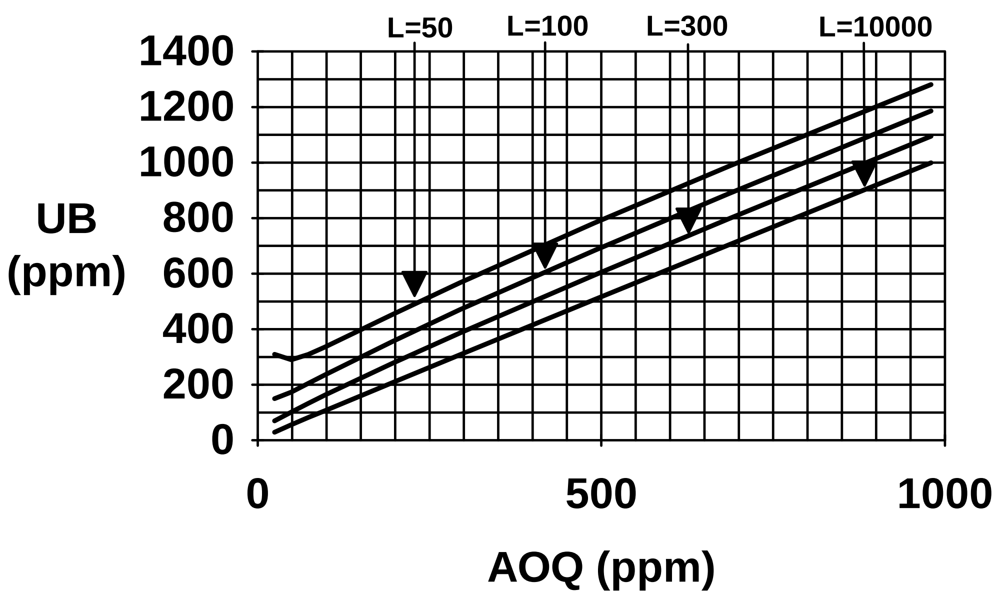

Figure B.4 — 90% Upper Bound, m=200

Annex C (informative) Derivation of the standard deviation of AOQ

Consider an arbitrary discrete-valued random variable ***X***. Denote the probability that ***X*** = x_i by Pr(***X*** = x_i), for I = 1, 2, 3,… . The set of values x_i for which Pr(***X*** = x_i) is positive may be either finite or infinite. If this set is finite, then there is some positive integer **J**, such that Pr(***X*** = x_i) = 0 whenever i>**J**.

The expected value of ***X*** is then,

$$\mathrm{E}(X) = \sum_{i=1}^{\infty} x_i \cdot \Pr(X = x_i)$$

and the expected value of $\boldsymbol{X}^2$ is,

$$\mathrm{E}(X^2) = \sum_{i=1}^{\infty} x_i^2 \cdot \Pr(X = x_i)\,.$$

The variance of ***X*** is,

$$\begin{aligned} \mathrm{V}(X) &= \mathrm{E}\left(\{X - [\mathrm{E}(X)]\}^2\right) \\ &= \mathrm{E}\left(X^2\right) - [\mathrm{E}(X)]^2 \end{aligned}$$

and the standard deviation of ***X***, denoted here as SD(***X***), is the square root of the variance of ***X***:

$$\mathrm{SD}(\boldsymbol{X}) = \sqrt{\mathrm{V}(\boldsymbol{X})}.$$

Annex C (informative) Derivation of the standard deviation of AOQ (cont'd)

The following notation is used in this annex. This list includes some symbols defined in section 5. In such cases, the definitions given here are consistent with those given in section 5, but may be stated here in more detail, for the purposes of this annex.

Symbol	Formula	Definition
AOQ'		Unknown true value of the average outgoing quality; the unknown true long-term average fraction nonconforming in outgoing product, expressed in parts per million nonconforming.
AOQ		A random variable to be used as an estimator for AOQ'.
AOQ		A numerical estimate of AOQ', calculated from sample data; a specific realization of the random variable ***AOQ***.
c		Sample plan acceptance number.
L		Total number of lots inspected.
LR		A random variable representing the total number of lots rejected out of the L lots inspected.
LR		Total number of lots actually rejected out of the L lots inspected; a specific realization of the random variable ***LR***.
LAR	1–(***LR***/L)	A random variable representing the lot acceptance rate.
LAR	1–(LR/L)	Lot acceptance rate actually achieved; a specific realization of the random variable ***LAR***.
m	S/L	Average sample size per inspected lot.
N		Total number of devices in all inspected lots.
D		Total number of nonconforming devices actually found from all samples; a specific realization of the random variable ***D***.
S	m x L	Total number of devices sampled from all inspected lots.
p'		Unknown true value of the fraction of nonconforming devices in all lots entering inspection for the first time.

Annex C (informative) Derivation of the standard deviation of AOQ (cont'd)

Symbol	Formula	Definition
$\boldsymbol{p}$	$\boldsymbol{D}/\mathrm{S}$	A random variable representing the fraction of nonconforming devices found in all samples.
p	D/S	Fraction of nonconforming devices found in all samples; a specific realization of the random variable $\boldsymbol{p}$.
$\boldsymbol{d}_{\mathrm{i}}$		A random variable representing the number of nonconforming devices in sample i.
d_{i}		Number of nonconforming devices actually occurring and found in sample i; a specific realization of the random variable $\boldsymbol{d}_{\mathrm{i}}$.
$\boldsymbol{A}_{\mathrm{i}}$		A random indicator function equal to one if lot i is accepted (i.e., if $\boldsymbol{d}_{\mathrm{i}} \leq c$) and equal to zero if lot i is rejected (i.e., if $\boldsymbol{d}_{\mathrm{i}} > c$).
θ_1	$\sum_{k=0}^{c} k \cdot \Pr(d_{\mathrm{i}} = k)$	$E(\boldsymbol{d}_{\mathrm{i}}\boldsymbol{A}_{\mathrm{i}})$; also, $E(\boldsymbol{d}_{\mathrm{i}}\boldsymbol{A}_{\mathrm{i}}^2)$.
θ_2	$\sum_{k=0}^{c} k^2 \cdot \Pr(d_{\mathrm{i}} = k)$	$E(\boldsymbol{d}_{\mathrm{i}}^2\boldsymbol{A}_{\mathrm{i}})$; also, $E(\boldsymbol{d}_{\mathrm{i}}^2\boldsymbol{A}_{\mathrm{i}}^2)$.
γ_1	$\sum_{k=0}^{\infty} k \cdot \Pr(d_{\mathrm{i}} = k)$	$E(\boldsymbol{d}_{\mathrm{i}})$
γ_2	$\sum_{k=0}^{\infty} k^2 \cdot \Pr(d_{\mathrm{i}} = k)$	$E(\boldsymbol{d}_{\mathrm{i}}^2)$
ψ	$\sum_{k=0}^{c} \Pr(d_{\mathrm{i}} = k)$	Probability that lot i is accepted, i.e., $\Pr(\boldsymbol{d}_{\mathrm{i}} \leq c)$; this is also equal to $E(\boldsymbol{A}_{\mathrm{i}})$ and to $E(\boldsymbol{A}_{\mathrm{i}}^2)$.

NOTE θ_1, θ_2, γ_1, γ_2, and ψ are independent of i; that is, each of these quantities is a fixed constant for all samples (equivalently, for all inspected lots).

Annex C (informative) Derivation of the standard deviation of AOQ (cont'd)

The formulas given for θ_1 and θ_2 are found as follows:

$$\begin{aligned}
\theta_1 &= E(\boldsymbol{d}_i\boldsymbol{A}_i) = E(\boldsymbol{d}_i\boldsymbol{A}_i^2)\\
&= [0\cdot 1\cdot \Pr(\boldsymbol{d}_i = 0) + 1\cdot 1\cdot \Pr(\boldsymbol{d}_i = 1) + \cdots + c\cdot 1\cdot \Pr(\boldsymbol{d}_i = c)]\\
&+ [(c{=}1)\cdot 0\cdot \Pr(\boldsymbol{d}_i = c+1) + (c+2)\cdot 0\cdot \Pr(\boldsymbol{d}_i = c+2) + \cdots]\\
&= \sum_{k=0}^{c} k\cdot \Pr(\boldsymbol{d}_i = k)
\end{aligned}$$

$$\begin{aligned}
\theta_2 &= E(\boldsymbol{d}_i^2\boldsymbol{A}_i) = E(\boldsymbol{d}_i^2\boldsymbol{A}_i^2)\\
&= [0^2\cdot 1\cdot \Pr(\boldsymbol{d}_i = 0) + 1^2\cdot 1\cdot \Pr(\boldsymbol{d}_i = 1) + \cdots + c^2\cdot 1\cdot \Pr(\boldsymbol{d}_i = c)]\\
&+ [(c{=}1)^2\cdot 0\cdot \Pr(\boldsymbol{d}_i = c+1) + (c+2)^2\cdot 0\cdot \Pr(\boldsymbol{d}_i = c+2) + \cdots]\\
&= \sum_{k=0}^{c} k^2\cdot \Pr(d_i = k)
\end{aligned}$$

The random variable ***AOQ*** is an estimator for the unknown value AOQ' and may be written as:

$$\begin{aligned}
\boldsymbol{AOQ} &= \boldsymbol{p}\cdot\boldsymbol{LAR}\cdot 10^6\\
&= \left(\frac{1}{mL}\sum_{i=1}^{L} d_i\right)\left(\frac{1}{L}\sum_{j=1}^{L} A_j\right)(10^6)\\
&= \left(\frac{10^6}{mL^2}\right)\left(\sum_{i=1}^{L} d_i\right)\left(\sum_{j=1}^{L} A_j\right)\\
&= \left(\frac{10^6}{mL^2}\right)\left(\sum_{i=1}^{L} d_i A_i + \sum_{\substack{i=1\\ i\neq j}}^{L}\sum_{j=1}^{L} d_i A_i\right)
\end{aligned}$$

Then the expected value of ***AOQ*** is found as follows, by using the statistical independence of $\boldsymbol{d}_i$ and $\boldsymbol{A}_j$ when $i \neq j$:

$$\begin{aligned}
E(\boldsymbol{AOQ}) &= \left(\frac{10^6}{mL^2}\right)\left[\sum_{i=1}^{L} E(\boldsymbol{d}_i\boldsymbol{A}_i) + \sum_{\substack{i=1\\ i\neq j}}^{L}\sum_{j=1}^{L} E(\boldsymbol{d}_i)\cdot E(\boldsymbol{A}_j)\right]\\
&= \left(\frac{10^6}{mL^2}\right)\left[L\theta_1 + L(L-1)\gamma_1\psi\right]\\
&= \left(\frac{10^6}{mL}\right)\left[(L-1)\psi\gamma_1 + \theta_1\right] \text{ ppm.}
\end{aligned}$$

Annex C (informative) Derivation of the standard deviation of AOQ (cont'd)

The square of ***AOQ*** is:

$$(AOQ)^2 = \left(\frac{10^{12}}{m^2L^4}\right)\left(\sum_{\alpha=1}^{L} d_\alpha\right)^2\left(\sum_{i=1}^{L} A_i\right)^2$$

$$= \left(\frac{10^{12}}{m^2L^4}\right)\left(\sum_{\alpha=1}^{L} d_\alpha^2 + \sum_{\substack{\beta=1 \\ \beta\neq\delta}}^{L}\sum_{\delta=1}^{L} d_\beta d_\delta\right)\cdot\left(\sum_{i=1}^{L} A_i^2 + \sum_{\substack{j=1 \\ j\neq k}}^{L}\sum_{k=1}^{L} A_j A_k\right)$$

$$= \left(\frac{10^{12}}{m^2L^4}\right)\left\{\left[\left(\sum_{\alpha=1}^{L} d_\alpha^2\right)\left(\sum_{i=1}^{L} A_i^2\right)\right] + \left[\left(\sum_{\alpha=1}^{L} d_\alpha^2\right)\left(\sum_{\substack{j=1 \\ j\neq k}}^{L}\sum_{k=1}^{L} A_j A_k\right)\right]\right.$$

$$\left. + \left[\left(\sum_{\substack{\beta=1 \\ \beta\neq\delta}}^{L}\sum_{\delta=1}^{L} d_\beta d_\delta\right)\left(\sum_{i=1}^{L} A_i^2\right)\right] + \left[\left(\sum_{\substack{\beta=1 \\ \beta\neq\delta}}^{L}\sum_{\delta=1}^{L} d_\beta d_\delta\right)\left(\sum_{\substack{j=1 \\ j\neq k}}^{L}\sum_{k=1}^{L} A_j A_k\right)\right]\right\}$$

Annex C (informative) Derivation of the standard deviation of AOQ (cont'd)

$$
\begin{aligned}
&= \left(\frac{10^{12}}{m^2L^4}\right)\Bigg\{\Bigg[\underset{(\alpha = i)}{\sum_{\alpha=1}^{L} d_\alpha^2 A_\alpha^2} + \underset{\alpha \neq i}{\sum_{\alpha=1}^{L}\sum_{i=1}^{L} d_\alpha^2 A_i^2}\Bigg] \\
&+\Bigg[\underset{\substack{\alpha \neq k \\ (\alpha = j)}}{\sum_{\alpha=1}^{L}\sum_{k=1}^{L} d_\alpha^2 A_\alpha A_k} + \underset{\substack{\alpha \neq j \\ (\alpha = k)}}{\sum_{\alpha=1}^{L}\sum_{j=1}^{L} d_\alpha^2 A_j A_\alpha} + \underset{\alpha \neq j \neq k \neq \alpha}{\sum_{\alpha=1}^{L}\sum_{j=1}^{L}\sum_{k=1}^{L} d_\alpha^2 A_j A_k}\Bigg] \\
&+\Bigg[\underset{\substack{i \neq \delta \\ (i = \beta)}}{\sum_{i=1}^{L}\sum_{\delta=1}^{L} d_i d_\delta A_i^2} + \underset{\substack{i \neq \beta \\ (i = \delta)}}{\sum_{i=1}^{L}\sum_{\beta=1}^{L} d_\beta d_i A_i^2} + \underset{i \neq \beta \neq \delta \neq i}{\sum_{\beta=1}^{L}\sum_{\delta=1}^{L}\sum_{i=1}^{L} d_\beta d_\delta A_i^2}\Bigg] \\
&+\Bigg[\underset{\substack{\beta \neq \delta \\ (\beta = j, \delta = k)}}{\sum_{\beta=1}^{L}\sum_{\delta=1}^{L} d_\beta d_\delta A_\beta A_\delta} + \underset{\substack{j \neq \beta \neq \delta \neq j \\ (\delta = k)}}{\sum_{\beta=1}^{L}\sum_{\delta=1}^{L}\sum_{j=1}^{L} d_\beta d_\delta A_j A_\delta} \\
&+\underset{\substack{\beta \neq \delta \\ (\beta = k, \delta = j)}}{\sum_{\beta=1}^{L}\sum_{\delta=1}^{L} d_\beta d_\delta A_\delta A_\beta} + \underset{\substack{k \neq \beta \neq \delta \neq k \\ (\delta = j)}}{\sum_{\beta=1}^{L}\sum_{\delta=1}^{L}\sum_{k=1}^{L} d_\beta d_\delta A_\delta A_k} \\
&+\underset{\substack{j \neq \beta \neq \delta \neq j \\ (\beta = k)}}{\sum_{\beta=1}^{L}\sum_{\delta=1}^{L}\sum_{j=1}^{L} d_\beta d_\delta A_j A_\beta} + \underset{\substack{k \neq \beta \neq \delta \neq k \\ (\beta = j)}}{\sum_{\beta=1}^{L}\sum_{\delta=1}^{L}\sum_{k=1}^{L} d_\beta d_\delta A_\beta A_k} \\
&+\underset{\substack{\beta \neq \delta \neq j \neq k \\ j \neq \beta \neq k \neq \delta}}{\sum_{\beta=1}^{L}\sum_{\delta=1}^{L}\sum_{j=1}^{L}\sum_{k=1}^{L} d_\beta d_\delta A_j A_k}\Bigg]\Bigg\}
\end{aligned}
$$

Annex C (informative) Derivation of the standard deviation of AOQ (cont'd)

$$= \left(\frac{10^{12}}{m^2L^4}\right)\left\{\sum_{\alpha=1}^{L} d_\alpha^2 A_\alpha^2 + \sum_{\alpha=1}^{L}\sum_{i=1}^{L} d_\alpha^2 A_i^2\right. \quad \alpha \neq i$$

$$+2\sum_{\alpha=1}^{L}\sum_{k=1}^{L} d_\alpha^2 A_\alpha A_k + \sum_{\alpha=1}^{L}\sum_{j=1}^{L}\sum_{k=1}^{L} d_\alpha^2 A_j A_k \quad \alpha \neq k \quad \alpha \neq j \neq k \neq \alpha$$

$$+2\sum_{i=1}^{L}\sum_{\delta=1}^{L} d_i d_\delta A_i^2 + \sum_{\beta=1}^{L}\sum_{\delta=1}^{L}\sum_{i=1}^{L} d_\beta d_\delta A_i^2 \quad i \neq \delta \quad i \neq \beta \neq \delta \neq i$$

$$+2\sum_{\beta=1}^{L}\sum_{\delta=1}^{L} d_\beta d_\delta A_\beta A_\delta + 4\sum_{\beta=1}^{L}\sum_{\delta=1}^{L}\sum_{j=1}^{L} d_\beta d_\delta A_j A_\delta \quad \beta \neq \delta \quad j \neq \beta \neq \delta \neq j$$

$$\left. + \sum_{\beta=1}^{L}\sum_{\delta=1}^{L}\sum_{j=1}^{L}\sum_{k=1}^{L} d_\beta d_\delta A_j A_k \right\} \quad \beta \neq \delta \neq j \neq k \quad j \neq \beta \neq k \neq \delta$$

Then the expected value of $(\boldsymbol{AOQ})^2$ is found as follows, again making use of statistical independence wherever possible:

$E[(AOQ)^2]$

$$= \left(\frac{10^{12}}{m^2L^4}\right)\left\{\sum_{\alpha=1}^{L} E(d_\alpha^2 A_\alpha^2) + \sum_{\alpha=1}^{L}\ \sum_{i=1}^{L} E(d_\alpha^2)\cdot E(A_\alpha^2)\right. \quad \alpha \neq j$$

Annex C (informative) Derivation of the standard deviation of AOQ (cont'd)

$$+2\sum_{\substack{\alpha=1\\ \alpha\neq k}}^{L}\sum_{k=1}^{L}E\left(d_\alpha^2 A_\alpha\right)\cdot E\left(A_k\right)+\sum_{\substack{\alpha=1\\ \alpha\neq j\neq k\neq\alpha}}^{L}\sum_{j=1}^{L}\sum_{k=1}^{L}E\left(d_\alpha^2\right)\cdot E\left(A_j\right)\cdot E\left(A_k\right)$$

$$+2\sum_{\substack{i=1\\ i\neq\delta}}^{L}\sum_{\delta=1}^{L}E\left(d_i A_i^2\right)\cdot E\left(d_\delta\right)+\sum_{\substack{\beta=1\\ i\neq\beta\neq\delta\neq i}}^{L}\sum_{\delta=1}^{L}\sum_{i=1}^{L}E\left(d_\beta\right)\cdot E\left(d_\delta\right)\cdot E\left(A_i^2\right)$$

$$+2\sum_{\substack{\beta=1\\ \beta\neq\delta}}^{L}\sum_{\delta=1}^{L}E\left(d_\beta A_\beta\right)\cdot E\left(d_\delta A_\delta\right)+4\sum_{\substack{\beta=1\\ j\neq\beta\neq\alpha\neq j}}^{L}\sum_{\delta=1}^{L}\sum_{j=1}^{L}E\left(d_\beta\right)\cdot E\left(d_\delta A_\delta\right)\cdot E\left(A_j\right)$$

$$+\sum_{\substack{\beta=1\\ \beta\neq\delta\neq j\neq k\\ j\neq\beta\neq k\neq\delta}}^{L}\sum_{\delta=1}^{L}\sum_{j=1}^{L}\sum_{k=1}^{L}E\left(d_\beta\right)\cdot E\left(d_\delta\right)\cdot E\left(A_j\right)\cdot E\left(A_k\right)\Big\}$$

$$\begin{aligned}=\left(\frac{10^{12}}{m^2L^4}\right)\Big\{&L\theta_2+L(L-1)\gamma_2\psi\\&+2L(L-1)\theta_2\psi+L(L-1)(L-2)\gamma_2\psi^2\\&+2L(L-1)\theta_1\gamma_1+L(L-1)(L-2)\gamma_1^2\psi\\&+2L(L-1)\theta_1^2+4L(L-1)(L-2)\gamma_1\theta_1\psi\\&+L(L-1)(L-2)(L-3)\gamma_1^2\psi^2\Big\}\end{aligned}$$

$$\begin{aligned}=\left(\frac{10^{12}}{m^2L^3}\right)\Big(&\theta_2+(L-1)\Big\{\left[2\psi\theta_2+2\gamma_1\theta_1+2\theta_1^2+\gamma_2\psi\right]\\&+(L-2)\psi\left[\left(4\gamma_1\theta_1+\gamma_2\psi+\gamma_1^2\right)+(L-3)\gamma_1^2\psi\right]\Big\}\Big)\end{aligned}$$

The square of the expected value of ***AOQ*** is

$$\left[E(AOQ)\right]^2=\left(\frac{10^{12}}{m^2L^2}\right)\left[(L-1)^2\psi^2\gamma_1^2+2(L-1)\psi\gamma_1\theta_1+\theta_1^2\right]$$

Annex C (informative) Derivation of the standard deviation of AOQ (cont'd)

The variance of ***AOQ*** is then:

$$
\begin{aligned}
\mathrm{V}(\boldsymbol{AOQ}) &= \mathrm{E}\left[(AOQ)^2\right]-\left[\mathrm{E}(AOQ)\right]^2 \\
&= \left(\frac{10^{12}}{\mathrm{m}^2\mathrm{L}^3}\right)(\theta_2+(\mathrm{L}-1)(2\psi\theta_2+2\gamma_1\theta_1+\gamma_2\psi) \\
&+ (\mathrm{L}-2)(\theta_1^2+(\mathrm{L}-1)(\mathrm{L}-2)\psi(\gamma_2\psi+\gamma_1^2) \\
&+ 2(\mathrm{L}-1)(\mathrm{L}-4)\psi\gamma_1\theta_1-2(2\mathrm{L}-3)(\mathrm{L}-1)\gamma_1^2\psi^2
\end{aligned}
$$

This expression for V(***AOQ***) is completely general — it applies with any appropriate probability distribution for $\boldsymbol{d}_\mathrm{i}$ and with any appropriate sample plan acceptance number c (i.e., any nonnegative integer c). Once the probability distribution for $\boldsymbol{d}_\mathrm{i}$ and the sample plan acceptance number c are identified, the variance of ***AOQ*** can be expressed in terms of m, p', and L. An estimate of the variance of ***AOQ*** can then be calculated by substituting the value of p for p' , along with the values of m and L. The square root of this estimate then provides an estimate of the standard deviation of ***AOQ***, in "units" of parts per million (ppm).

In this document it is assumed, as an approximation, that $\boldsymbol{d}_\mathrm{i}$ follows a Poisson probability distribution with mean mp'. Therefore,

$$\Pr(\boldsymbol{d}_\mathrm{i}=\mathrm{k})=\frac{e^{-mp'}(mp')^k}{k!} \quad \text{for } k=0,1,2,\ldots$$

$$\theta_1=\sum_{k=0}^{c} k\,\frac{e^{-mp'}(mp')^k}{k!}$$

$$\theta_2=\sum_{k=0}^{c} k^2\,\frac{e^{-mp'}(mp')^k}{k!}$$

$$\gamma_1=\mathrm{E}(\boldsymbol{d}_\mathrm{i})=mp'$$

$$\gamma_2=\mathrm{E}(\boldsymbol{d}_\mathrm{i}^2)=(mp')(1+mp')$$

$$\psi=\sum_{k=0}^{c}\frac{e^{-mp'}(mp')^k}{k!}$$

Annex C (informative) Derivation of the standard deviation of AOQ (cont'd)

Sampling plans with c = 0 lead to

$\theta_1 = \theta_2 = 0$ and $\psi = e^{-mp'}$

so V(***AOQ***) simplifies to

$$V(AOQ) \cong \left(\frac{10^{12}}{mL^3}\right)(L-1)p'e^{-mp'}\left\{1+(L-1)mp'+\left[(L-2)-(3L-4)mp'\right]e^{-mp'}\right\}$$

This expression can be approximated by a series expansion in p', dropping terms of order $(p')^3$ or higher. This results in the variance formula of equation (2), reproduced below:

$$V(AOQ) \cong \left(\frac{10^{12}}{mL^3}\right)(L-1)p'\left\{(L-1)-2(2L-3)mp'\right\}$$

Sampling plans with c = 1 lead to

$\theta_1 = \theta_2 = mp'e^{-mp'}$ and $\psi = (1 + mp')e^{-mp'}$

so V(***AOQ***) simplifies to

$$\begin{aligned} V(AOQ) = \left(\frac{10^{12}}{mL^3}\right)p'e^{-mp'}\,&\{[L(L-1)+(4-6L+L(mp') \\ &-(3L+2)(L-1)(mp')^2-(3L-4)(L-1)(mp')^3]e^{-mp'} \\ &+L+(L-1)(mp')+(L-1)^2(mp')^2\} \end{aligned}$$

This expression also can be approximated by a series expansion in p', dropping terms of order $(p')^3$ or higher. This results in the approximation

$$V(AOQ) \cong \left(\frac{10^{12}}{mL^3}\right)(p')\,\{(L^2-2)(2L-1)(mp')\}$$

Annex D (informative) References

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Annex E (informative) Differences between JESD16B and JESD16A

This table briefly describes most of the changes made to entries that appear in this standard, JESD16B, compared to its predecessor, JESD16-A (April 1995).

Clause	**Description of change**
Intro and throughout	Changed “component(s)” to “device(s)”.
4	Paragraph numbering removed from terms and definitions section.
4	Inserted note under **acceptance inspection.**
4	Minor editorial changes to definition of **average outgoing quality (AOQ).**
4	Added definition for acronym “ppm” for the term **class.**
4	For the term **class**, examples became a NOTE and were re-ordered.
4	Minor edit to the term **fraction nonconforming** and the second sentence became a note.
4	Second sentence in the definition of **inspection** became a NOTE.
4	Corrected the definition of **lot acceptance rate (LAR).**
4	Second sentence in the definition of **nonconformity** became a NOTE.
4	Added “or lot acceptance rate (LAR)” to the definition of **sample period.**
5	Added “per lot” to the definition of Symbol m.
6.2	Corrected standard references.
6.4	Deleted “final” in the first sentence (referred to acceptance inspection).
6.7	Added “upon request”.
6.12	This paragraph added.
7.1	Edited for clarity.
7.3.1	Changed “group” to “collection” and added NOTE. Added “+” between last two terms.
7.5	Added reference to an example in Annex A.
7.6	Edited for clarity.
7.7	Deleted primes from two instances of “AOQ” where it was included erroneously. Corrected paragraph reference. Deleted “where the quantity exp(t) is e^{t}” after eq. 7.
7.7.1	Corrected “graph” to “Figure”.
7.7.2	Changed “can” to “may”. Changed “graph” to “Figure”. Inserted “numeric” in the second paragraph. Changed annex reference from A to B in second paragraph.
8.2	Minor editorial changes.
8.3	Changed equation reference number from 8 to 11.
8.3.1	Changed equation number from 8 to 11.
Annex D	Updated references.

Standard Improvement Form JEDEC JESD16B

The purpose of this form is to provide the Technical Committees of JEDEC with input from the industry regarding usage of the subject standard. Individuals or companies are invited to submit comments to JEDEC. All comments will be collected and dispersed to the appropriate committee(s).

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1. I recommend changes to the following:
   - [ ] Requirement, clause number ________
   - [ ] Test method number ________ Clause number ________

   The referenced clause number has proven to be:
   - [ ] Unclear [ ] Too Rigid [ ] In Error
   - [ ] Other ________

2. Recommendations for correction:

3. Other suggestions for document improvement:

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